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THE ADDITION OF QUASI-THREE-DIMENSIONAL TERMS INTO A FINITE ELEMENT METHOD FOR TRANSONIC TURBOMACHINERY BLADE-TO-BLADE FLOWS

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SUMMARY

This paper describes the extension of a purely two-dimensional finite element method for the calculation of transonic turbomachinery blade-to-blade flows to include the quasi-three-dimensional terms. These terms account for the effect of variations in streamline radius, stream-tube height and blade rotation.

By approximating the stream surface as a piecewise linear function, then using a local developed cone transformation on an element basis, the finite element equations are shown to remain of the same form as the two-dimensional equations.

The numerical results presented demonstrate that the stream-tube height, streamline radius and blade rotation terms must be included if the prediction of the Mach number distribution around a gas turbine blade is to be calculated correctly.

KEY WORDS Turbomachines Finite Elements Transonic Flows

INTRODUCTION

A commonly adopted procedure for the design of three-dimensional turbomachinery blades is that first proposed by Wu[1] involving two quasi-three-dimensional programs, namely a through-flow program together with a blade-to-blade program. The through-flow provides inlet and exit design conditions, e.g. Mach number and whirl angle, together with the streamline radius and stream-tube height variations. The individual blade sections are designed using a blade-to-blade method and then stacked to give a three-dimensional blade.

It is known that the effects of stream-tube height variations through the blade row are important and that the effects of streamline radius variations are important for rotating blade rows; these effects must be included in any practical blade-to-blade method.

A number of blade-to-blade methods exist based mainly on finite difference techniques, e.g. streamline curvature, matrix methods and time-marching. More recently finite element methods have been applied to the problem. These have been shown to be both economical and versatile in terms of the grid resolution that is possible. One area of importance is the blade leading edge where a rapid acceleration takes place and the geometry can have an important effect on the boundary layer behaviour.

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The two-dimensional method described by Whitehead and Grant[2] using a velocity potential has been shown to be fast and accurate.[3] In Reference 3 a full description of the method and the boundary conditions is given. It is shown that by using the upwinding technique or artificial compressibility described in Reference 3, the method can handle transonic and supersonic flows with shocks.

This paper details the quasi-three-dimensional extensions to the method to include the effects of streamline radius, stream-tube height and rotation. It is shown that for a general axisymmetric stream-surface a local developed cone analysis can be adopted on an element basis. In the case of small elements and/or small cone angles the analysis relates closely to that for a two-dimensional plane surface.

VELOCITY POTENTIAL FORMULATION OF THE QUASI-THREE-DIMENSIONAL BLADE-TO-BLADE EQUATIONS

The quasi-three-dimensional blade-to-blade equations are obtained from the full three-dimensional equations by integrating from one axisymmetric stream-surface to a neighbouring one. The continuity equation for steady flow in a relative co-ordinate system rotating with the blade row can be written as

$$\frac{\partial}{\partial m}(hR\rho W_m) + \frac{\partial}{\partial \theta}(h\rho W_\theta) = 0 \tag{1}$$

where W_m is the relative meridional velocity, W_θ the relative whirl velocity and ρ the density. The directions m, R and θ, and the stream-tube height h are defined in Figure 1.

If it is assumed that rothalpy and entropy are constant on the stream surface, then Crocco's equation shows that the flow is irrotational in an absolute frame of reference. Because of this a

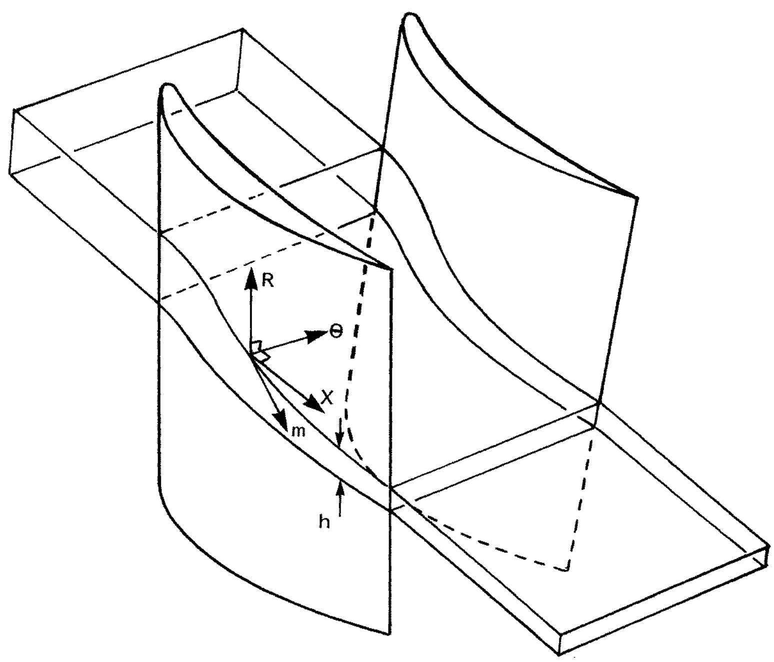

Figure 1. Quasi-3D stream surface

velocity potential (ϕ) can be introduced, the absolute velocity ($\mathbf{q}$) being defined as

$$\mathbf{q} = \nabla\phi \tag{2}$$

where

$$\nabla = \left(\frac{\partial}{\partial m}, \frac{1}{R}\frac{\partial}{\partial \theta}\right)$$

The relative velocity is related to the velocity potential by

$$\mathbf{W} = \nabla\phi - \mathbf{\Omega} \times \mathbf{R} \tag{3}$$

where $\mathbf{\Omega}$ is the rotational speed of the blades. The continuity equation may now be written in terms of velocity potential as

$$\frac{\partial}{\partial m}\left(\rho Rh\frac{\partial\phi}{\partial m}\right) + \frac{\partial}{R\partial\theta}\left(\rho Rh\left(\frac{\partial\phi}{R\partial\theta} - \Omega R\right)\right) = 0 \tag{4}$$

As rothalpy is constant throughout the flow field,

$$C_p t + \tfrac{1}{2}\mathbf{q}^2 - \Omega R q_\theta = \text{constant} \tag{5}$$

where C_p is the specific heat at constant pressure, t is the static temperature and q_θ is the absolute whirl velocity.

By assuming that the flow is that of a perfect adiabatic gas and relating the conditions to those on the inlet plane the density can be expressed as

$$\rho = \rho_{0_{IN}}\left[1 - \frac{\gamma - 1}{2C_{0_{IN}}^2}(\mathbf{W}^2 - \Omega^2(R^2 - R_{IN}^2))\right]^{1/(\gamma-1)} \tag{6}$$

where $\rho_{0_{IN}}$ and $C_{0_{IN}}$ are, respectively, the relative stagnation density and sound speed at inlet.

This equation shows the effect of blade rotation on the relationship between density and velocity on a stream surface of varying radius.

SOLUTION PROCEDURE

The velocity/density relationship (6) means that the continuity equation (4) is non-linear in velocity potential. Because of this a Newton–Raphson technique is used to solve the problem. At any stage a current approximation to the solution is denoted by an overbar. The difference between this and the correct solution is denoted by a prime, and it is assumed that these perturbations are small. Hence

$$\begin{aligned} \phi &= \bar{\phi} + \phi' \\ \rho &= \bar{\rho} + \rho' \\ q_i &= \bar{q}_i + q_i' \end{aligned} \tag{7}$$

Substituting into equation (1) and neglecting products of primed quantities gives

$$\nabla\cdot h(\rho'\bar{\mathbf{W}} + \bar{\rho}\mathbf{W}') = -\nabla\cdot(h\bar{\rho}\bar{\mathbf{W}}) \tag{8}$$

Similarly expanding equation (6) and neglecting second and higher order terms gives

$$\rho' = -\frac{\bar{\rho}}{\bar{C}^2}\bar{\mathbf{W}}\cdot\mathbf{W}' \tag{9}$$

where

$$\mathbf{W}' = \nabla\phi'$$

FINITE ELEMENT DISCRETIZATION

To obtain the discrete finite element equations the solution domain is first divided into elements over which the variations of the unknowns are prescribed in terms of nodal values using shape functions. In the present analysis the Galerkin weighted residual method is used.

In a two-dimensional Cartesian co-ordinate system the division of the solution domain into elements is relatively easy, one of the simplest elements being a straight-sided triangle over which the velocity potential is assumed to vary linearly in the two co-ordinates. The linear variation of velocity potential results in the velocity, and therefore density, being constant over an element. The evaluation of integrals over the element is therefore trivial.[2,3] On a general axisymmetric stream surface the definition of an equivalent element is more complicated. Any three nodes on the surface would have to be joined by curves lying in the surface. The area of the stream surface bounded by such an element becomes complex to calculate. In addition integrals over the element are further complicated in the case of rotating blades, as both the relative velocity and density will vary with streamline radius over an element. Appendix I shows that if the streamline radius is defined from a through-flow analysis as a piecewise linear function along the machine axis (x) the stream surface can be approximated locally by part of a cone (Figure 8). This allows a local transformation onto a developed cone to be performed in order to define straight-sided triangular elements. The velocity potential varies linearly within each element on the transformed surface and it can be shown that the absolute velocity components will be constant over each element. However, the relative whirl velocity (W_θ) and consequently the density vary as functions of streamline radius. In Appendix I it is shown that if the change in streamline radius across an element is small, the expressions for element areas, shape functions and velocity components are the same as those for a two-dimensional plane surface but using m and $R\theta$ as co-ordinates. Therefore the shape functions may be written as

$$\phi \approx a + bm + cR\theta \tag{10}$$

Applying the Galerkin weighted residual method to equation (8) for an internal node j gives

$$\sum_{n=1}^{N(j)} \int_{A_n} h(\rho'\bar{\mathbf{W}} + \bar{\rho}\mathbf{W}')\cdot\nabla Z_{n(j)}\mathrm{d}A = -\sum_{n=1}^{N(j)} \int_{A_n} \bar{\rho}h\bar{\mathbf{W}}\cdot\nabla Z_{n(j)}\,\mathrm{d}A \tag{11}$$

where the summation is over every element containing the internal node (j) and $Z_{n(j)}$ is the shape function corresponding to element j. If the velocity and density perturbations are written in terms of a perturbation to velocity potential then (dropping the overbar notation)

$$\sum_{n=1}^{N(j)} \int_{A_n} \rho_n h_n \phi'_p \left[\frac{\partial Z_p}{\partial x_i}\frac{\partial Z_{n(j)}}{\partial x_i} - \frac{W_l W_i}{C_n^2}\frac{\partial Z_p}{\partial x_l}\frac{\partial Z_{n(j)}}{\partial x_i}\right]_n \mathrm{d}A = \sum_{n=1}^{N(j)} \int_{A_n} \rho_n h_n \left[W_i \frac{\partial Z_{n(j)}}{\partial x_i}\right]_n \mathrm{d}A \tag{12}$$

where

$$\frac{\partial}{\partial x_1} \equiv \frac{\partial}{\partial m}, \frac{\partial}{\partial x_2} \equiv \frac{1}{R}\frac{\partial}{\partial \theta}$$

With blade rotation and variations in stream-tube height and streamline radius across an element an integration over an element would be complex. In Appendix II it is shown that if the variations in stream-tube height and streamline radius across an element are small then an accurate approximation to equation (12) is

$$\sum_{n=1}^{N(j)} \tilde{\rho}_n \tilde{h}_n A_n \phi'_p \left[\frac{\partial Z_p}{\partial x_i}\frac{\partial Z_{n(j)}}{\partial x_i} - \frac{\tilde{W}_l \tilde{W}_i}{\tilde{C}_n^2}\frac{\partial Z_p}{\partial x_l}\frac{\partial Z_{n(j)}}{\partial x_i}\right]_n = -\sum_{n=1}^{N(j)} \tilde{\rho}_n \tilde{h}_n A_n \left[\tilde{W}_i \frac{\partial Z_{n(j)}}{\partial x_i}\right]_n \tag{13}$$

Where $\tilde{\rho}_n$, $\tilde{h}_n$, $\tilde{W}_n$ and $\tilde{C}_n$ are quantities evaluated at the centroid of the element.

The system of equations formed from (13) at each node gives a linear set of equations to be solved for ϕ', the correction to velocity potential at each node. This forms an iterative procedure in which ϕ is replaced by $\phi + \phi'$ on each iteration until convergence is achieved.

This solution scheme converges for subsonic flows, but for transonic and supersonic flows it becomes unstable. In order to overcome this an artificial compressibility technique is adopted where the density used in each element is 'upwinded' if the local Mach number is supersonic. A full description of the method is given by Whitehead and Newton.[3]

NUMERICAL RESULTS

In Reference 3 the ability of the method to model transonic and supersonic flows with shocks is demonstrated. The following examples have been included to validate the quasi-three-dimensional extension to the method.

(i) Turbine examples

Figure 2 depicts the finite element mesh generated automatically to analyse the flow in a turbine cascade.[4,5] The cascade has flared endwalls resulting in a linear variation of stream-tube height,

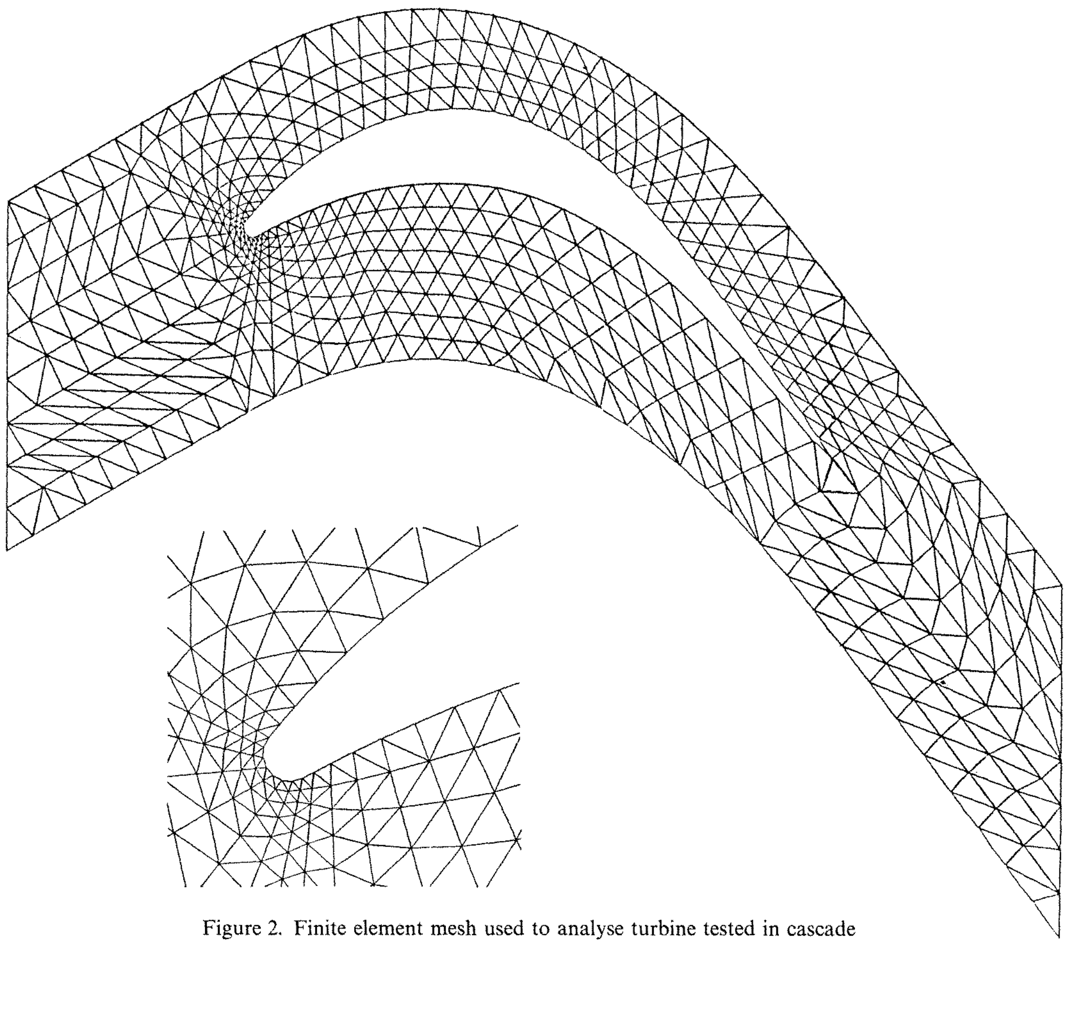

Figure 2. Finite element mesh used to analyse turbine tested in cascade

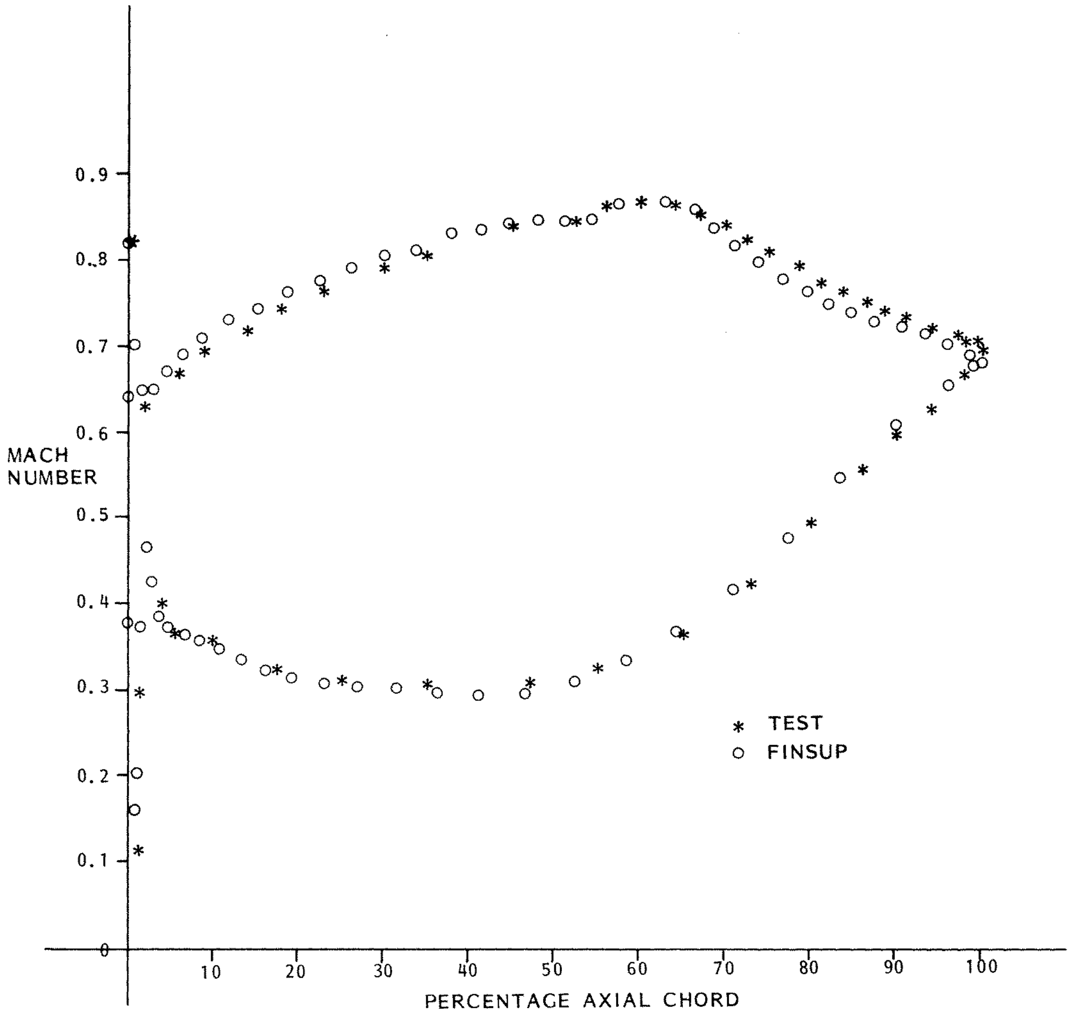

Figure 3. Comparison of predicted and measured Mach number distribution around turbine blade tested in cascade (inlet Mach number = 0·5, inlet angle = 38·8°)

$h_2/h_1 = 1{\cdot}095$. Figure 3 is a comparison of the Mach number distributions calculated using the finite element program FINSUP with experimental results. This shows very good agreement, the main area of difference being near the trailing edge on the suction surface. This is probably due to viscous effects not modelled in this version of FINSUP.

Comparisons have been made using FINSUP and a streamline curvature method proven at Rolls-Royce for a turbine rotor of similar geometry to the one tested in cascade. Figure 4 shows the close agreement of the Mach number distributions where no variations in stream-tube or streamline radius are included. The Mach number distribution shown in Figure 5 is for the same blade and inlet conditions but with the inclusion of a variation in stream-tube height ($h_2/h_1 = 1{\cdot}091$) and streamline radius ($R_2/R_1 = 1{\cdot}015$) obtained from a through-flow analysis. The blade has a rotation speed of 456·6 rad/s. Again excellent agreement between the two methods is obtained. A comparison of Figures 4 and 5 show that the addition of the quasi-three-dimensional terms has greatly changed the exit conditions from the blade row and thus the work performed.

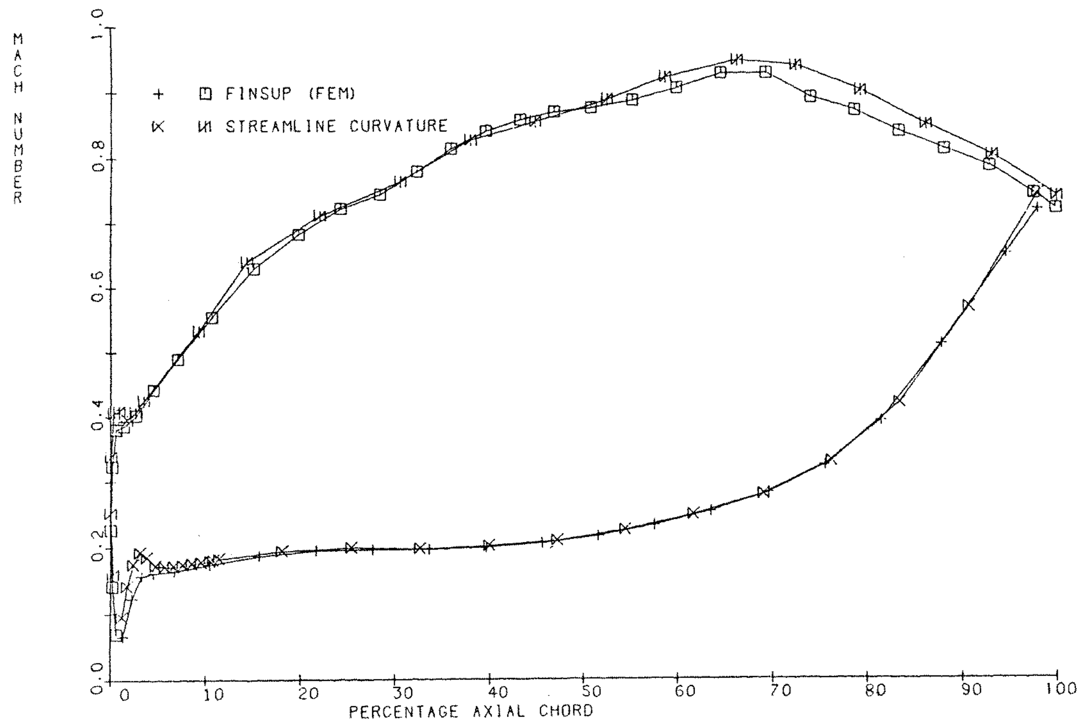

Figure 4. Comparison of FINSUP and a streamline curvature method for turbine blade on a 2D stream surface (inlet Mach number = 0·273, inlet angle = 16·1°)

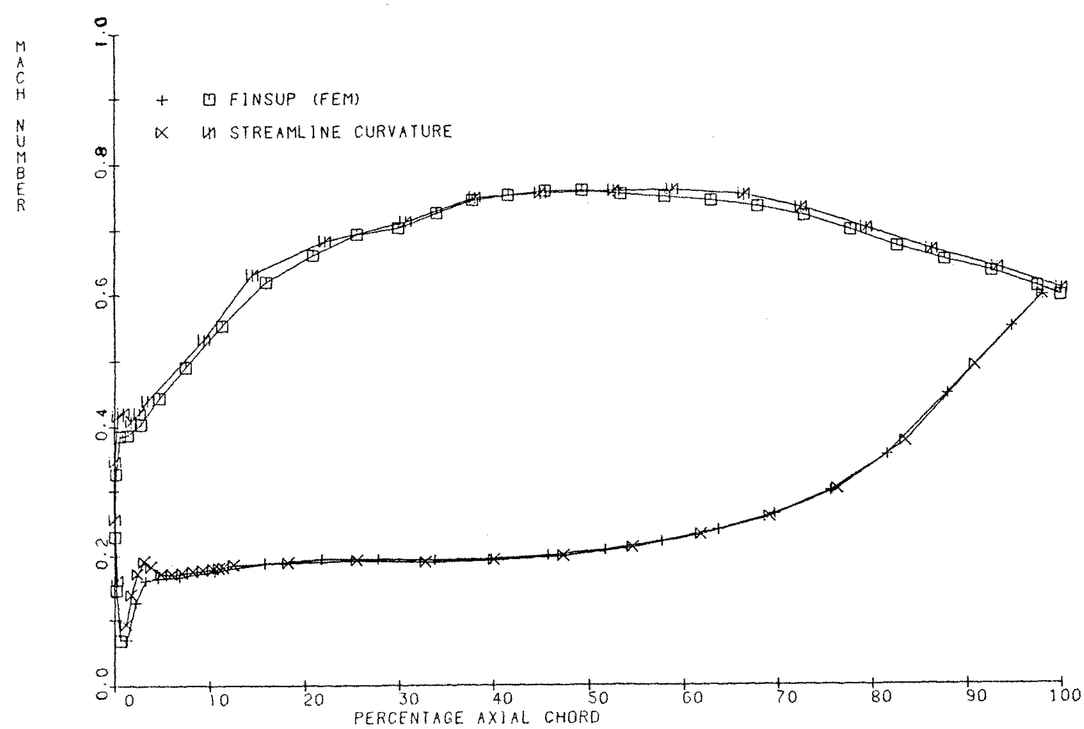

Figure 5. Comparison of FINSUP and a streamline curvature method for a turbine blade on a quasi 3D stream surface (inlet Mach number = 0·273, inlet angle = 16·1°)

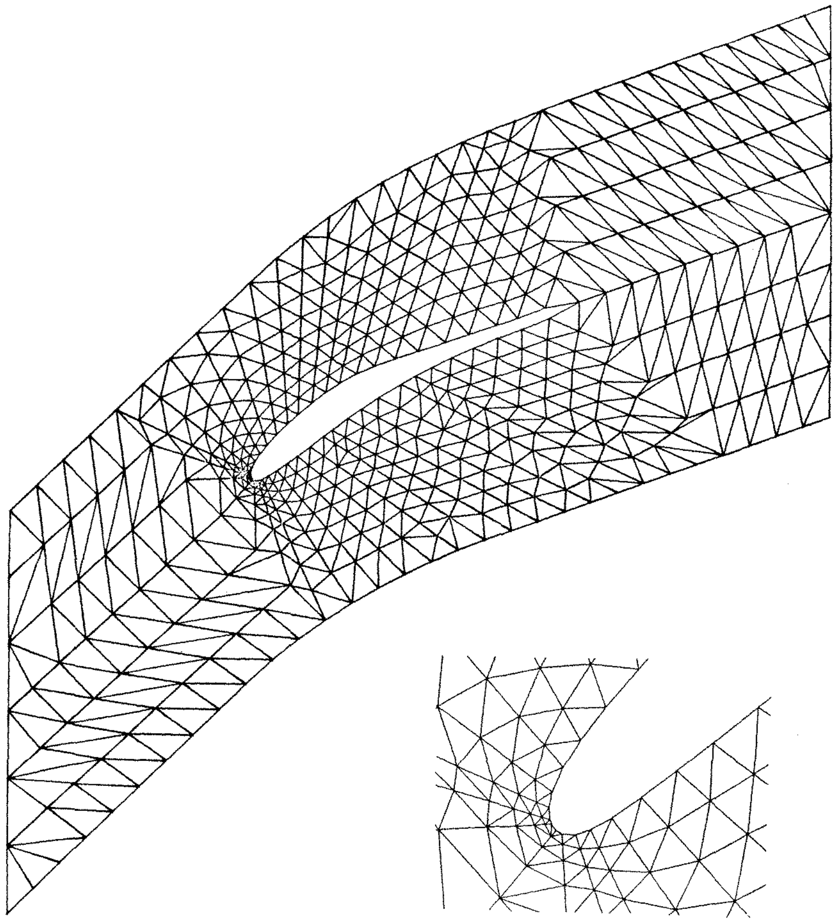

Figure 6. Finite element mesh used to analyse BGK test blade

(ii) Supercritical compressor

There is currently much interest in the design of highly loaded shockless supercritical compressor blades with controlled suction surface diffusion. The Bauer, Garabedian and Korn (BGK) hodograph method can be used for the design of such blades. It is however two-dimensional and, consequently, if used in a design procedure an analysis program is required to study the sensitivity of the blades to inlet conditions and the quasi-three-dimensional effects. Figure 6 depicts the mesh generated automatically using the geometry of the BGK test blade[6]. The calculated Mach number distribution (Figure 7) shows extremely good agreement with that used by the BGK method to produce the blades. The results are also shown for the same blade with a realistic variation of stream-tube height, streamline radius and blade rotation. It can be seen that the quasi-three-dimensional effects are significant and must be taken into account in a multi-stage compressor.

CONCLUSIONS

A finite element method for the calculation of transonic turbomachinery flows has been extended from a purely two-dimensional calculation, to include the effect of variations in streamline radius and stream-tube height, and blade rotation. By approximating the stream surface as a piecewise

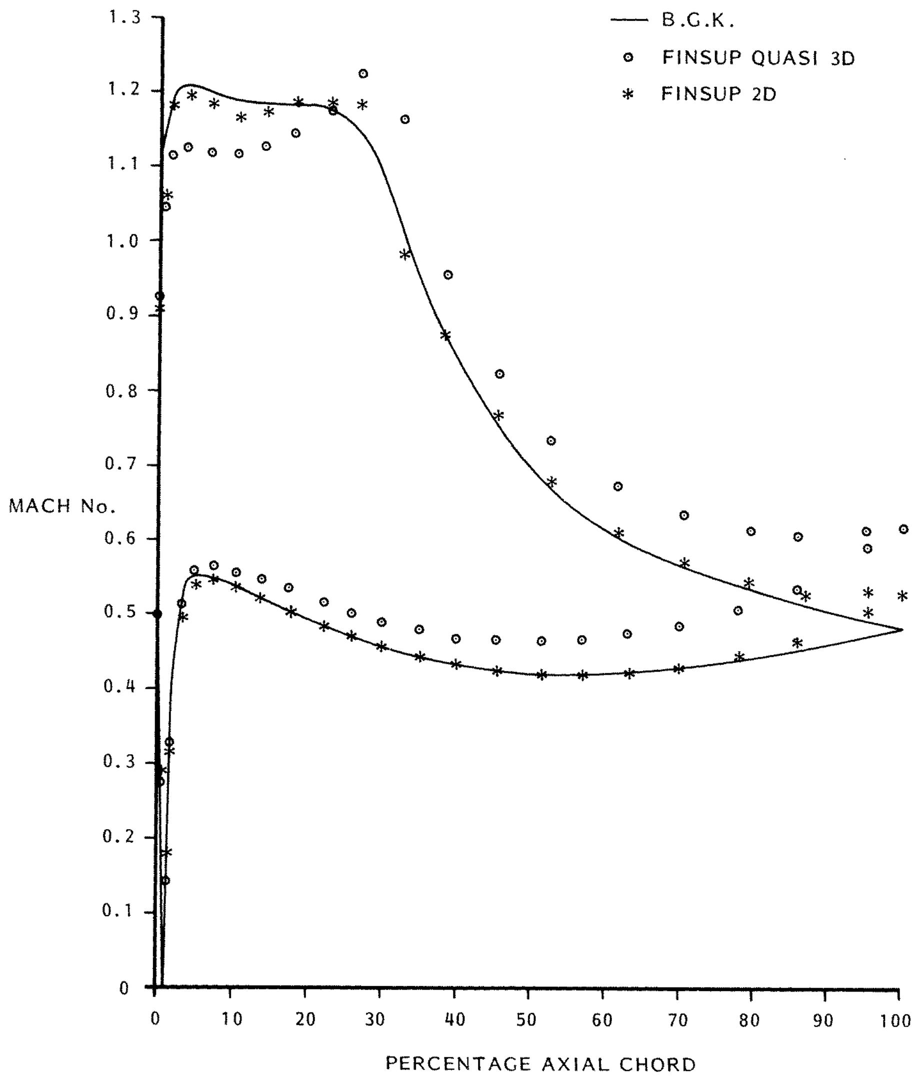

Figure 7. Predicted Mach number distribution around BGK test blade (inlet Mach number = 0·72, inlet angle = 45·9°)

linear function, then using a local developed cone transformation on an element basis the equations for element areas, shape functions and velocity components are shown to remain of the same form as for a two dimensional calculation. If the changes in stream-tube height and streamline radius across an element of the mesh are assumed to be small then the integrals occurring in the finite element analysis may be accurately approximated using element centroid values. In current blade designs this presents no limitations.

Numerical results have been presented showing the importance of the quasi-three-dimensional terms. These demonstrate that the stream-tube height, streamline radius and blade rotation terms must be included if the Mach number distribution around a gas turbine blade is to be predicted correctly.

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APPENDIX I. LOCAL DEVELOPED CONE ANALYSIS

It is assumed that the streamline radius is obtained from a through-flow calculation and is defined as a piecewise linear function in the direction of the machine axis (x). The stream surface covered by a curvilinear triangular element can therefore be approximated locally by part of a cone (Figure 8), the cone semi-angle (γ) being determined from the stream-surface definition and meridional co-ordinate.

For two nodes A and B on the stream-surface

$$m_{\mathrm{B}} = m_{\mathrm{A}} = \int_{\mathrm{A}}^{\mathrm{B}} \sqrt{\left[1 + \left(\frac{\mathrm{d}R}{\mathrm{d}x}\right)^2\right]} \mathrm{d}x \tag{14}$$

If A and B lie on one linear part of the stream-surface then

$$m_{\mathrm{B}} - m_{\mathrm{A}} = m_{\mathrm{B}}^* - m_{\mathrm{A}}^* \tag{15}$$

where m^* is measured along a generator of the local approximating cone from the apex. The local developed cone transformation is applied to each element in terms of co-ordinates m^* and ψ, where ψ is defined by

$$m^*\psi = R(\theta - \theta_{\mathrm{A}}) \tag{16}$$

A being a node of the element (Figure 9).

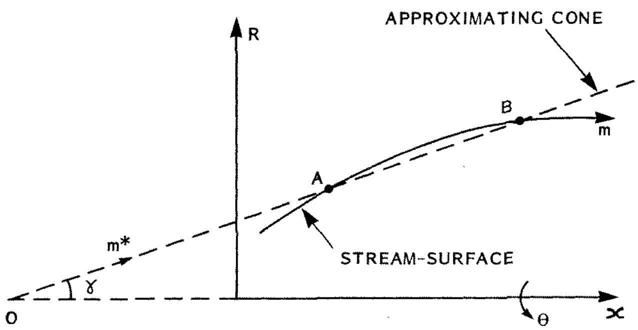

Figure 8. Approximation of stream surface locally by part of a cone

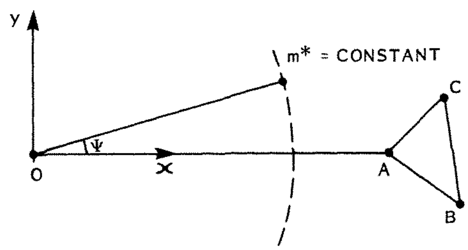

Figure 9. Development of a local cone

Elements

The transformation maps nodes A, B and C of the elements onto the development of the local cone. In this plane the sides of the element are straight lines. It is convenient to introduce a local Cartesian co-ordinate system defined by

$$\begin{aligned} x &= m^* \cos\psi \\ y &= m^* \sin\psi \end{aligned} \tag{17}$$

The area of triangular element ABC is

$$A = \tfrac{1}{2}\mathrm{Det}\begin{bmatrix} 1 & 1 & 1 \\ x_A & x_B & x_C \\ y_A & y_B & y_C \end{bmatrix} \tag{18}$$

For small elements and/or small cone semi-angles differences in ψ over an element will be small. Using equation (16) it can be shown that

$$A \approx \tfrac{1}{2}\mathrm{Det}\begin{bmatrix} 1 & 1 & 1 \\ m_A & m_B & m_C \\ (R\theta)_A & (R\theta)_B & (R\theta)_C \end{bmatrix} \tag{19}$$

This is the same as in a strictly two-dimensional analysis but using m and $R\theta$ as co-ordinates.

Velocity components

Over an element the velocity potential is bilinear, i.e.

$$\phi = a + bx + cy \tag{20}$$

In terms of nodal values

$$\phi = \sum_{i=1}^{3} \phi_i Z_i(x, y)$$

where Z_i, the shape function, is linear in x and y. In an element

$$\begin{aligned} \frac{\partial\phi}{\partial x} &= \sum_i \phi_i \frac{\partial Z_i}{\partial x} = \frac{1}{2A}\sum_i \phi_i(y_j - y_k) = \text{constant} \\ \frac{\partial\phi}{\partial y} &= \sum_i \phi_i \frac{\partial Z_i}{\partial y} = \frac{-1}{2A}\sum_i \phi_i(x_j - x_k) = \text{constant} \end{aligned} \tag{21}$$

The absolute meridional and whirl velocities are defined as

$$\begin{aligned} q_m &= \left(\frac{\partial\phi}{\partial m}\right)_\theta \\ q_\theta &= \frac{1}{R}\left(\frac{\partial\phi}{\partial\theta}\right)_m \end{aligned} \tag{22}$$

For small elements and/or small cone semi-angles it can be shown that

$$q_m \approx \frac{1}{2A}\sum_i \phi_i[(R\theta)_j - (R\theta)_k - (m_j^* - m_k^*)\theta\sin\gamma] \tag{23}$$

In the applications considered

$$(m_j^* - m_k^*)\theta \sin\gamma \ll (R\theta)_j - (R\theta)_k \tag{24}$$

so that

$$q_m \approx \frac{1}{2A}\sum_i \phi_i((R\theta)_j - (R\theta)_k) = \text{constant} \tag{25}$$

Similarly

$$q_\theta \approx \frac{-1}{2A}\sum_i \phi_i(m_j - m_k) = \text{constant} \tag{26}$$

Summary

(i) By approximating the streamline radius by a piecewise linear function the axisymmetric stream surface is approximated by series of cones.
(ii) For elements a local developed cone transformation can be adopted in order to define elements, calculate areas and shape functions.
(iii) It is shown that for small elements and/or small cone semi-angles the expressions for element areas and absolute velocity components are the same as those for a two-dimensional plane surface but using m and $R\theta$ as co-ordinates. In particular the meridional and absolute whirl velocity components are constant over an element.

APPENDIX II. USE OF QUANTITIES EVALUATED AT THE ELEMENT CENTROIDS

In the application of the Galerkin weighted residual method to the stream-tube averaged continuity equation (12) it is necessary to evaluate integrals of the forms

$$I_1 = \int_A \rho h W_i \frac{\partial Z_{n(j)}}{\partial x_i} \mathrm{d}A \tag{27}$$

$$I_2 = \int_A \rho h \phi' \left[\frac{\partial Z_P}{\partial x_i}\frac{\partial Z_{n(j)}}{\partial x_i} - \frac{W_l W_i}{C_n^2}\frac{\partial Z_P}{\partial x_l}\frac{\partial Z_{n(j)}}{\partial x_i} \right] \tag{28}$$

From Appendix I

$$\begin{aligned} W_m &= q_m = \text{constant} \\ W_\theta &= q_\theta - \Omega R = \text{constant} - \Omega R \end{aligned} \tag{29}$$

The variation of density with streamline radius is given by equation (6). In order to perform the integration an expansion technique about the value at the centroid of the element is used. The streamline radius is written as

$$R = \tilde{R} + R^* \tag{30}$$

where $\tilde{R}$ is the radius at the centroid of the element. The value of the element mean radius is used to define mean values of W_θ and ρ as follows

$$W_\theta = \tilde{W}_\theta(1 + W_\theta^*) \tag{31}$$

where

$$\tilde{W}_\theta = \frac{-1}{2A}\sum_i \phi_i(m_j - m_k) - \Omega\tilde{R} = \text{constant} \tag{32}$$

and

$$W_\theta^* = \frac{-\Omega R^*}{\tilde{W}_\theta} = \text{constant} \times R^* \tag{33}$$

In a similar fashion

$$\rho = \tilde{\rho}(1 + \rho^*) \tag{34}$$

where, from equation (6)

$$\tilde{\rho} = \rho_{0_{\mathrm{IN}}}\left[1 - \frac{\gamma - 1}{2C_{0_{\mathrm{IN}}}^2}(\mathbf{W}^2 - \Omega^2(\tilde{R}^2 - R_{\mathrm{IN}}^2))\right]^{1/(\gamma-1)} = \text{constant} \tag{35}$$

and

$$\rho^* = \frac{\dfrac{\Omega^2 \tilde{R} R^*}{C_{0_{\mathrm{IN}}}^2} + \text{higher order terms}}{1 - \dfrac{\gamma - 1}{2C_{0_{\mathrm{IN}}}^2}(\mathbf{W}^2 - \Omega^2(\tilde{R}^2 - R_{\mathrm{IN}}^2))} \approx \text{constant} \times R^* \tag{36}$$

where it has been assumed that R^* is small in the expansion of equation (6). As $\partial Z_i/\partial m$ and $\partial Z_i/R\partial\theta$ are constant over an element (see Appendix I) we can write

$$I_1 = \tilde{I}_1 + I_1^* \tag{37}$$

where

$$\tilde{I}_1 = \left[\tilde{\rho}\tilde{h}\left(W_m \frac{\partial Z_i}{\partial m} + \tilde{W}_\theta \frac{1}{R}\frac{\partial Z_i}{\partial \theta}\right)\right] \tag{38}$$

and

$$I_1^* = \tilde{\rho}\tilde{h}\left[W_m \frac{\partial Z_i}{\partial m}\int_A \left(\rho^* + \frac{h^*}{\tilde{h}}\right)\mathrm{d}A + \tilde{W}_\theta \frac{1}{R}\frac{\partial Z_i}{\partial \theta}\int_A \left(\rho^* + \frac{h^*}{\tilde{h}} + W_\theta^*\right)\mathrm{d}A\right] \tag{39}$$

Since R and h are assumed to be piecewise linear then

$$I_1^* = 0 \tag{40}$$

to the order considered. This means that the integral I_1, in equation (27) can be evaluated using quantities at the centroid of the element.

A similar analysis would show that I_2 in equation (28) can also be evaluated using quantities at the centroid. It can be shown that the neglected terms in the expansion are proportional to $(m - \tilde{m})^2\ (\mathrm{d}h/\mathrm{d}m)^2$ and $(m - \tilde{m})^2\ \mathrm{d}R/\mathrm{d}m \cdot \mathrm{d}h/\mathrm{d}m$, both of which in general will be very small.

Summary

In the case of no rotation, the meridional and whirl velocity components and density are constant over an element. In evaluating the integrals over an element, the stream-tube height can be replaced by the value at the centroid of the element.

In the case with rotation, the meridional component of velocity is constant over an element, but both whirl velocity and density vary. In the evaluation of the integrals over an element, quantities can be replaced by their values calculated at the centroid of the elements.

It should be noted that the assumptions made in Appendix I of changes in stream-tube height and streamline radius being small over an element are also made in this analysis.

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